Finite Element Methods

Week No-03

Introduction to Matrix Structural Analysis

- Classical Methods vs. Matrix Methods.
- Planar Frame Member, Nodal Displacements & Nodal Forces in Local Coordinates
- Truss Analysis by Stiffness Matrix Method
  - Truss Member Stiffness Relations in Local Coordinates
  - Member Stiffness Relations in Global Coordinates
  - Structure Stiffness Relations
  - Procedure for Analysis
- Frame & Beam Analysis by Stiffness Matrix Method
  - Analytical Model for Planar Frame Structure
  - Global & Local Coordinate Systems
  - Member Stiffness Relations in Local Coordinates
  - Stiffness Matrix of 2D Frame Member in Global Coordinates
  - Member Stiffness Relations in Global Coordinates
  - Structure Stiffness Relations

Classical Methods vs Matrix Methods

Classical Methods	Matrix Methods
Help to understand the structural behavior & the principles of structural Analysis	Simplify the overall picture of Structural Analysis
Time consuming for the analysis of large systems	Time saving as being computerized
Vary according to the structure type	Less varying

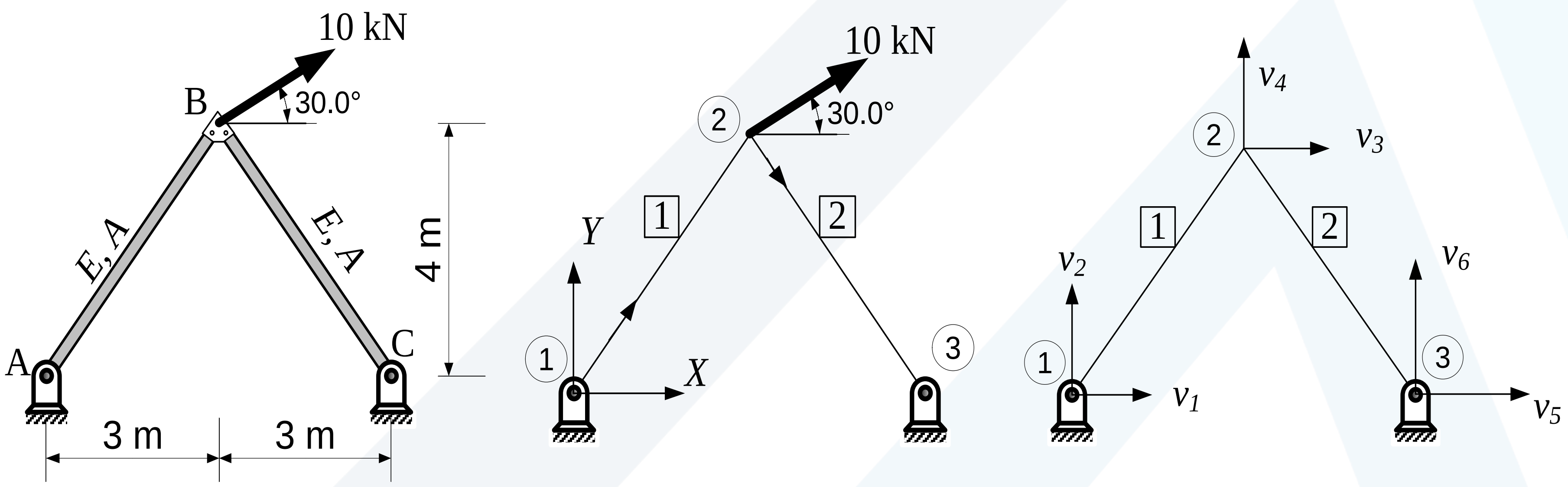
EXAMPLE . Element Stiffness Matrix

Figure shows a planar truss. The material is steel, E = 2(10)^{11}Pa and the cross-sectional area of member AB and BC are respectively 0.01 m^2 and 0.02 m^2.

1. Construct the global stiffness matrix for each member & the truss stiffness matrix, then find the displacement of joint B.
2. Find the member internal forces and the support reactions.

SOLUTION

Step 1: We select kN, m as the problem units. The origin of the global coordinate system will be located at point A. The labeled model is shown in Figs. and (c).



It helps to create the following table that contains the terms in the global element stiffness matrix.

EXAMPLE . Element Stiffness Matrix

Member	(X_b,Y_b)	(X_e,Y_e)	L	l	m	AE/L
1	(0,0)	(3,4)	5	0.6	0.8	4×10^5
2	(3,4)	(6,0)	5	0.6	-0.8	8×10^5

Step 2& 3: We now create the two global element stiffness matrices.

For element 1, we have:

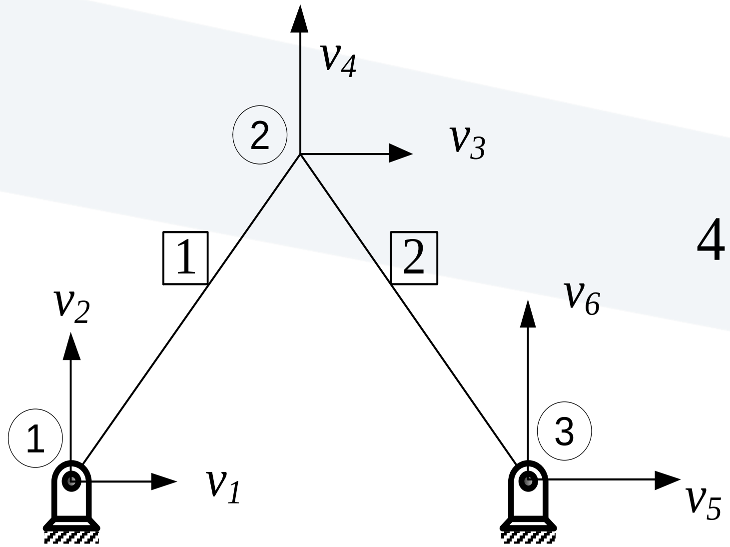
$$\mathbf{K}^1_{4\times4} = \frac{AE}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} = 4\times10^5\begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 \\ 0.48 & 0.64 & -0.48 & -0.64 \\ -0.36 & -0.48 & 0.36 & 0.48 \\ -0.48 & -0.64 & 0.48 & 0.64 \end{bmatrix}$$

For element 2, we have

$$\mathbf{K}^2_{4\times4} = \frac{AE}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} = 8\times10^5\begin{bmatrix} 0.36 & -0.48 & -0.36 & 0.48 \\ -0.48 & 0.64 & 0.48 & -0.64 \\ -0.36 & 0.48 & 0.36 & -0.48 \\ 0.48 & -0.64 & -0.48 & 0.64 \end{bmatrix}$$

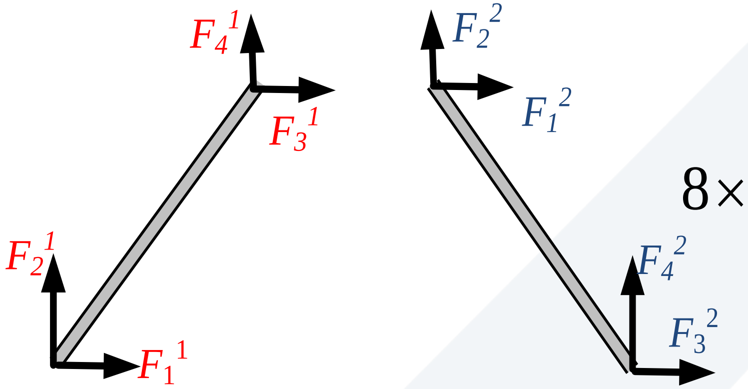
EXAMPLE . Element Stiffness Matrix

Finally, the global element equilibrium equations for the two elements can be written as



For Element 1

$$4\times10^{5}\begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 \\ 0.48 & 0.64 & -0.48 & -0.64 \\ -0.36 & -0.48 & 0.36 & 0.48 \\ -0.48 & -0.64 & 0.48 & 0.64 \end{bmatrix}\begin{Bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} F_1^1 \\ F_2^1 \\ F_3^1 \\ F_4^1 \end{Bmatrix}$$



For Element 2

$$8\times10^{5}\begin{bmatrix} 0.36 & -0.48 & -0.36 & 0.48 \\ -0.48 & 0.64 & 0.48 & -0.64 \\ -0.36 & 0.48 & 0.36 & -0.48 \\ 0.48 & -0.64 & -0.48 & 0.64 \end{bmatrix}\begin{Bmatrix} v_3 \\ v_4 \\ v_5 \\ v_6 \end{Bmatrix} = \begin{Bmatrix} F_1^2 \\ F_2^2 \\ F_3^2 \\ F_4^2 \end{Bmatrix}$$

Useless equations: the two matrices are singular & the two nodal load columns are unknown

EXAMPLE . Element Stiffness Matrix

Step4. From the 8 equations of the 2 separated members, we need to construct the 6 equations describing the equilibrium of the entire truss. Starting with member 1.

$$4\times10^5\begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 & 0 & 0 \\ 0.48 & 0.64 & -0.48 & -0.64 & 0 & 0 \\ -0.36 & -0.48 & 0.36 & 0.48 & 0 & 0 \\ -0.48 & -0.64 & 0.48 & 0.64 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{Bmatrix}=\begin{Bmatrix} F_1^1 \\ F_2^1 \\ F_3^1 \\ F_4^1 \\ 0 \\ 0 \end{Bmatrix}$$

Now using the equations from member 2, we get

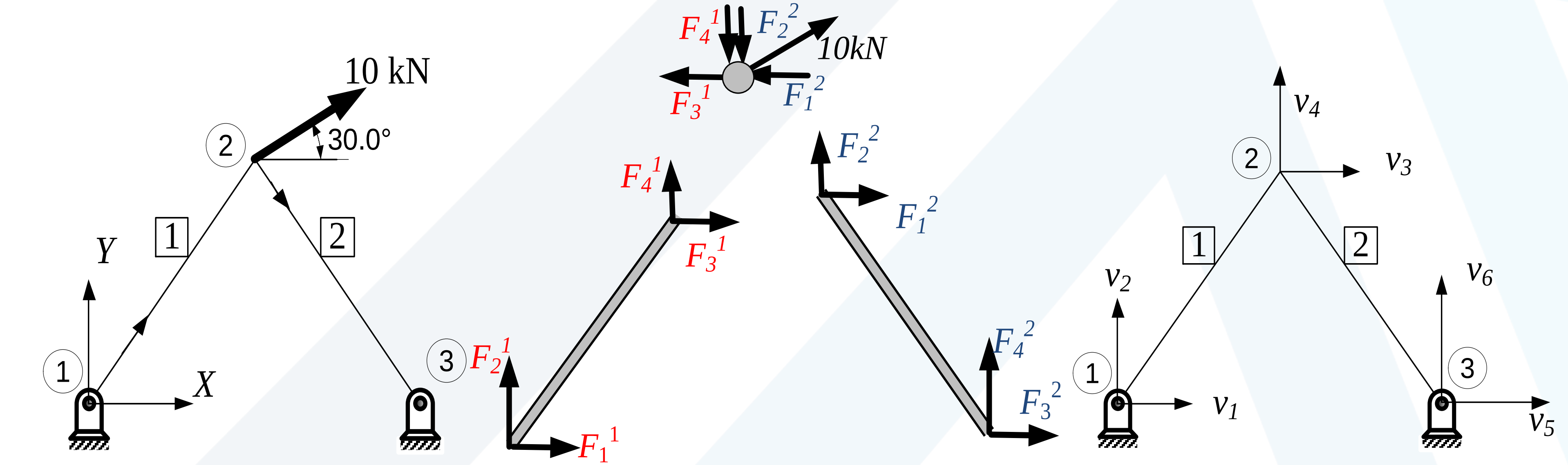
$$4\times10^5\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.72 & -0.96 & -0.72 & 0.96 \\ 0 & 0 & -0.96 & 1.28 & 0.96 & -1.28 \\ 0 & 0 & -0.72 & 0.96 & 0.72 & -0.96 \\ 0 & 0 & 0.96 & -1.28 & -0.96 & 1.28 \end{bmatrix}\begin{Bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ F_1^2 \\ F_2^2 \\ F_3^2 \\ F_4^2 \end{Bmatrix}$$

EXAMPLE . Element Stiffness Matrix

$$4\times10^{5}\begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 & 0 & 0 \\ 0.48 & 0.64 & -0.48 & -0.64 & 0 & 0 \\ -0.36 & -0.48 & 0.36+0.72 & 0.48-0.96 & -0.72 & 0.96 \\ -0.48 & -0.64 & 0.48-0.96 & 0.64+1.28 & 0.96 & -1.28 \\ 0 & 0 & -0.72 & 0.96 & 0.72 & -0.96 \\ 0 & 0 & 0.96 & -1.28 & -0.96 & 1.28 \end{bmatrix}\begin{Bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{Bmatrix}=\begin{Bmatrix} F_1^1 \\ F_2^1 \\ F_3^1+F_1^2 \\ F_4^1+F_2^2 \\ F_3^2 \\ F_4^2 \end{Bmatrix}$$

Elements within the rectangle show the coupling between the two truss members.

The component of the applied load at node 2 are *10cos(30°)* & *10sin(30°)*, we have the system equations as.



$$4\times10^5\begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 & 0 & 0 \\ 0.48 & 0.64 & -0.48 & -0.64 & 0 & 0 \\ -0.36 & -0.48 & 1.08 & -0.48 & -0.72 & 0.96 \\ -0.48 & -0.64 & -0.48 & 1.92 & 0.96 & -1.28 \\ 0 & 0 & -0.72 & 0.96 & 0.72 & -0.96 \\ 0 & 0 & 0.96 & -1.28 & -0.96 & 1.28 \end{bmatrix}\begin{Bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{Bmatrix}=\begin{Bmatrix} F_1 \\ F_2 \\ 8.66 \\ 5.00 \\ F_5 \\ F_6 \end{Bmatrix}$$

Symbolically

$$\mathbf{K}_{6\times6}\mathbf{v}_{6\times1}=\mathbf{F}_{6\times1}$$

We can carry out a few checks to ensure that the results are acceptable. First, the structural stiffness matrix **K** should be symmetric. Second, **K** is *usually* diagonally dominant, meaning that the diagonal element has the largest magnitude in that row and column. As we can see in the equations above, this condition is met by most but not all diagonal elements. However, the largest number is a diagonal element, K_{44}. Note also that all diagonal elements are positive.

Step 5: The boundary conditions for this problem are $v_1 = v_2 = v_5 = v_6 = 0$. Imposing these conditions on the system equations yields the modified system equations:

$$4\times10^5\begin{bmatrix} 1.08 & -0.48 \\ -0.48 & 1.92 \end{bmatrix}\begin{Bmatrix} v_3 \\ v_4 \end{Bmatrix}=\begin{Bmatrix} 8.66 \\ 5.00 \end{Bmatrix}$$

or symbolically $\mathbf{K}_{2\times2}\mathbf{v}_{2\times1}=\mathbf{F}_{2\times1}$

Step 6: Solve the system equations **Kv = F** for the nodal displacements **v.**

$v_3 = 2.581\times10^{-5}m$ & $v_4 = 1.296\times10^{-5}m$,

$v=(v_3^2+v_4^2)^{1/2}=2.888\times10^{-5}m$

Step 7: *Computing the internal forces .* For each element using the nodal displacements, compute the element nodal forces. We first start with local element matrix

$$\mathbf{Q}_{2\times1} = \mathbf{k}_{2\times2}\mathbf{u}_{2\times1}$$

Substituting from the kinematic global to local transformation equation, we have

$$\mathbf{Q}_{2\times1} = \mathbf{k}_{2\times2}\mathbf{T}_{2\times4}\mathbf{v}_{4\times1}$$

Since each element is in equilibrium $\quad Q_1 = -Q_4$

It is necessary to compute only one of the element local forces. For example,

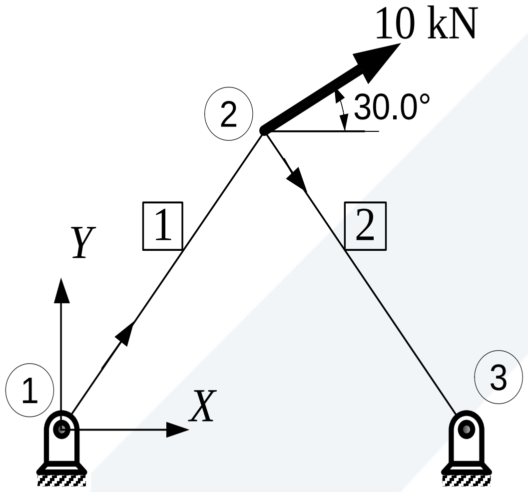
$$Q_1 = \frac{EA}{L}\begin{bmatrix} l & m & -l & -m \end{bmatrix}\begin{Bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{Bmatrix}$$

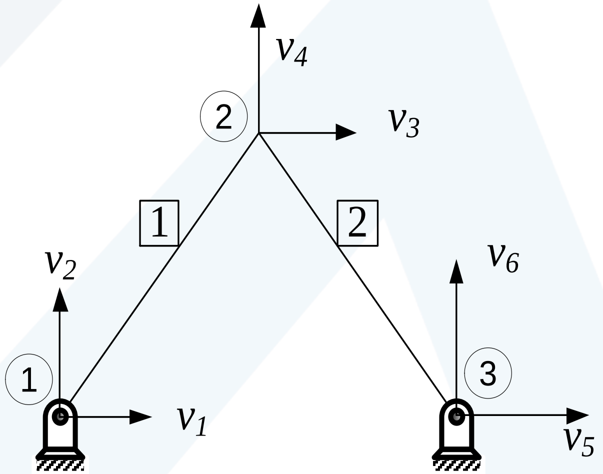
For element 1, we have

$$Q_1 = 4\times10^5\begin{bmatrix}0.6 & 0.8 & -0.6 & -0.8\end{bmatrix}\begin{bmatrix}0 & 0 & v_3 & v_4\end{bmatrix}^{\mathrm{T}} = -?????\ \mathrm{kN}\ (\mathrm{T\ or\ C})$$

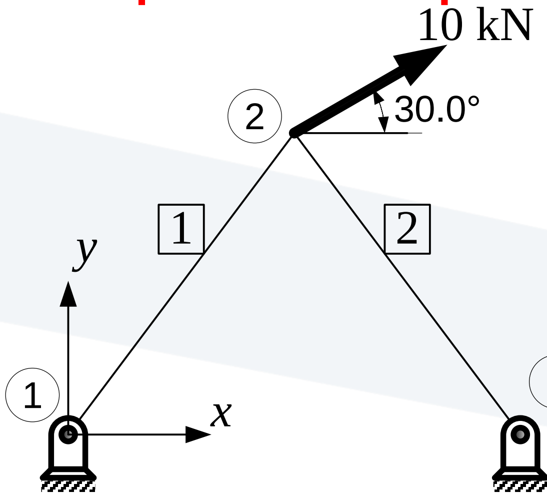
For member 2, we have

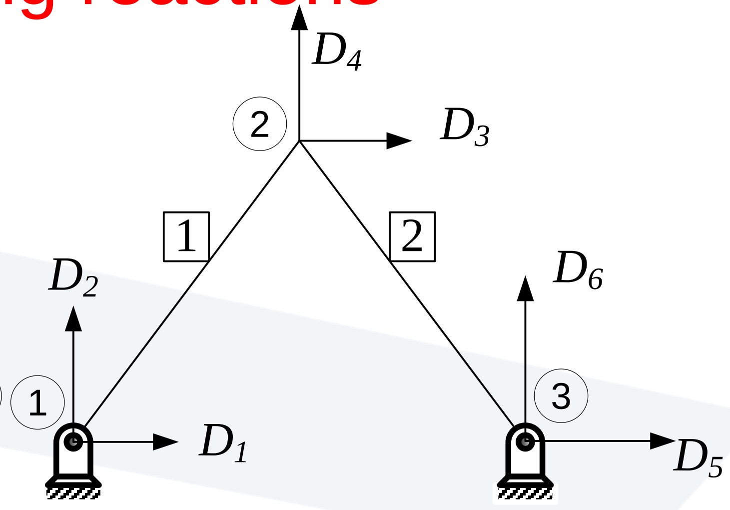
$$Q_1 = 8\times10^5\begin{bmatrix}0.6 & -0.8 & -0.6 & 0.8\end{bmatrix}\begin{bmatrix}v_3 & v_4 & 0 & 0\end{bmatrix}^{\mathrm{T}} = ??????\ \mathrm{kN}\ (\mathrm{T\ or\ C})$$

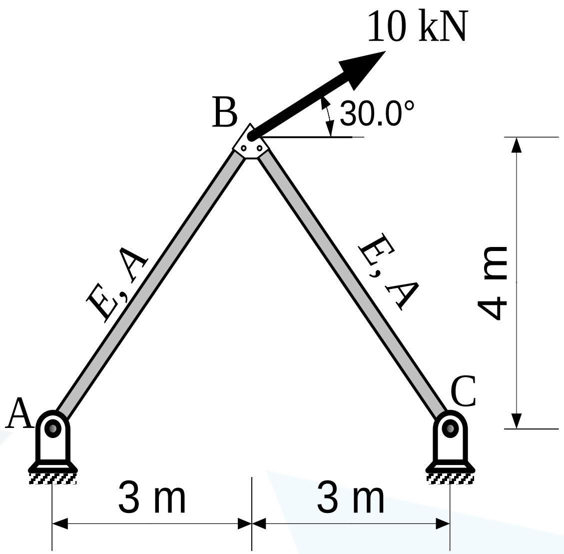




Step 8. Computing reactions







$$4\begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 & 0 & 0 \\ 0.48 & 0.64 & -0.48 & -0.64 & 0 & 0 \\ -0.36 & -0.48 & 1.08 & -0.48 & -0.72 & 0.96 \\ -0.48 & -0.64 & -0.48 & 1.92 & 0.96 & -1.28 \\ 0 & 0 & -0.72 & 0.96 & 0.72 & -0.96 \\ 0 & 0 & 0.96 & -1.28 & -0.96 & 1.28 \end{bmatrix}\begin{Bmatrix} v_1 \\ v_2 \\ 2.581 \\ 1.296 \\ v_5 \\ v_6 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ 8.66 \\ 5.00 \\ F_5 \\ F_6 \end{Bmatrix}$$

$$A_x = F_1 = 4[(-0.36)(2.581) + (-0.48)(1.296)] = ?????\ \text{kN}$$

$$C_x = F_5 = ????\ \text{kN}$$

$$A_y = F_2 = 4[(-0.48)(?????) + (-0.64)(??????)] = ?????\ \text{kN}$$

We summarize the major steps in solving a planar truss problem using the direct stiffness method.
Note that a truss element has two d.o.f in the local coordinate system and four d.o.f in the global coordinate system.

Step 1: Select the problem units & the global coordinate system. Identify and label the nodes and the elements. For each element select a start node (node 1) and an end node (node 2). Use an arrow along the member to indicate the direction from the start node to the end node. This establishes the local coordinate system for each element. Label the two global d.o.f at each node starting at node 1 and proceeding sequentially.

Step 2: Construct the equilibrium-compatibility equations for a typical element.

Step 3: Using the problem data, construct the element equations from ***Step 2*** for all the elements in the problem.

Step 4: Assemble the element equations into the system equations, $\mathbf{K}_{2Jx2J}\,\mathbf{V}_{2Jx1} = \mathbf{F}_{2Jxl}$, where J is the number of joints in the truss.

Step 5: Impose the boundary conditions.

Step 6: Solve the system equations **KD = F** for the nodal displacements **D.**

Step 7: For each element using the nodal displacements, compute the element nodal forces. We first start with local element matrix

$$\mathbf{Q}_{2\times1} = \mathbf{k}_{2\times2}\mathbf{u}_{2\times1}$$

Substituting from the kinematic local to global transformation equation, we have

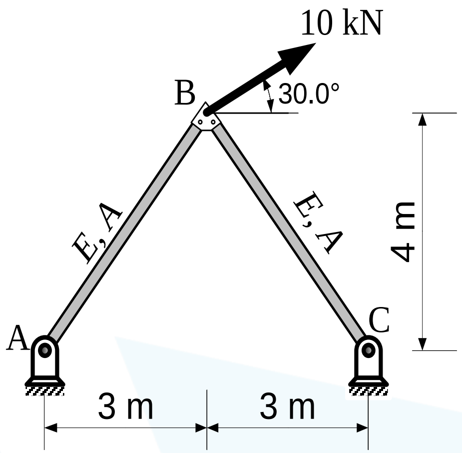
$$\mathbf{Q}_{2\times1} = \mathbf{k}_{2\times2}\mathbf{T}_{2\times4}\mathbf{v}_{4\times1}$$

Since each element is in equilibrium $\quad Q_1 = -Q_4$

It is necessary to compute only one of the element local forces. For example,

$$Q_1 = \frac{EA}{L}\begin{bmatrix} l & m & -l & -m \end{bmatrix}\begin{Bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{Bmatrix}$$

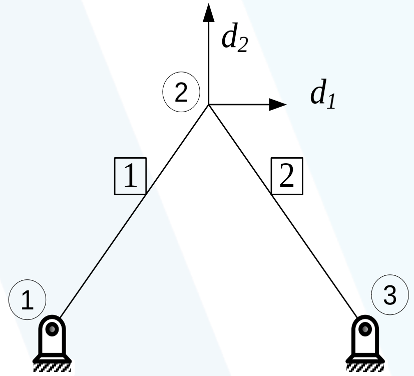
Example.1. For the given truss use the matrix method to.(1) Construct the Analytical model & determine the global degrees of freedom. (2) For each member construct the global stiffness matrix. (3) Then establish the truss stiffness matrix & solve for the global degrees of freedom.
(4) Finally Determine the member internal forces and the reactions.



1 - Analytical Model & Degrees of Freedom:

From the Analytical model in Fig. (b), the truss has only two d. o. f. d_1 & d_2, the global translations of joint 2.

$$\mathbf{d} = \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}$$



2 - Member Stiffness matrices:

a – Member1:

$$\mathbf{K}^1_{4\times4} = 4\times10^5 \begin{array}{c} \begin{array}{cccc} 0 & 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 \\ 0.48 & 0.64 & -0.48 & -0.64 \\ -0.36 & -0.48 & 0.36 & 0.48 \\ -0.48 & -0.64 & 0.48 & 0.64 \end{bmatrix} \end{array} \begin{array}{c} \\ 0 \\ 0 \\ 1 \\ 2 \end{array}$$

b– Member2:

$$\begin{array}{c} \\ \mathbf{K}^{2}_{4\times4} = 8\times10^{5} \end{array} \begin{array}{cccc|c} 1 & 2 & 0 & 0 & \\ \hline 0.36 & -0.48 & -0.36 & 0.48 & 1 \\ -0.48 & 0.64 & 0.48 & -0.64 & 2 \\ -0.36 & 0.48 & 0.36 & -0.48 & 0 \\ 0.48 & -0.64 & -0.48 & 0.64 & 0 \end{array}$$

3 – Structure (or Truss) Stiffness matrix **S** :
a- Member Matrices assembling

$$\mathbf{S} = 4\times10^{5} \begin{array}{cc} 1 & 2 \\ \end{array} \begin{bmatrix} (0.36+0.72) & (0.48-0.96) \\ (0.48-0.96) & (0.64+1.28) \end{bmatrix} \begin{array}{c} 1 \\ 2 \end{array} = 4\times10^{5} \begin{bmatrix} 1.08 & -0.48 \\ -0.48 & 1.92 \end{bmatrix} \begin{array}{c} 1 \\ 2 \end{array}$$

b - Solving for the global degrees of freedom

$$\bar{\mathbf{P}} = \begin{Bmatrix} 8.66 \\ 5 \end{Bmatrix}$$

$$\bar{\mathbf{P}} = \mathbf{S}\mathbf{d} \qquad \begin{Bmatrix} 8.66 \\ 5 \end{Bmatrix} = 4\times10^{5}\begin{bmatrix} 1.08 & -0.48 \\ -0.48 & 1.92 \end{bmatrix}\begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}$$

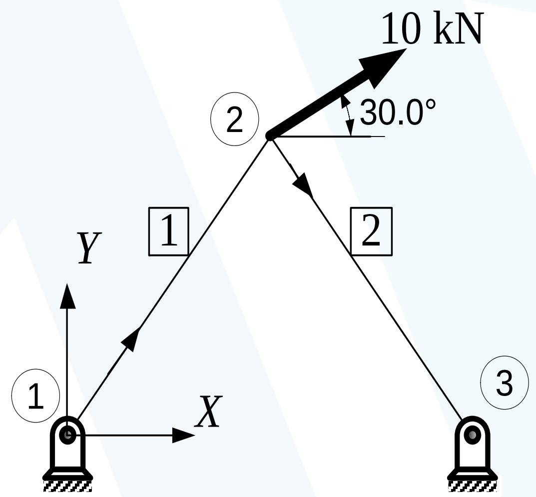
$$\mathbf{d} = \begin{Bmatrix} 2.581\times10^{-5}\,\mathrm{m} \\ 1.296\times10^{-5}\,\mathrm{m} \end{Bmatrix}$$

4 – Member End Displacements & End Forces:

a – Member1:

Global Displacement Vector

$$\mathbf{v}_1 = \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{Bmatrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 2 \end{matrix} = \begin{Bmatrix} 0 \\ 0 \\ d_1 \\ d_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 2.581\times10^{-5}\,\mathrm{m} \\ 1.296\times10^{-5}\,\mathrm{m} \end{Bmatrix}$$



Local Displacement Vector

$$\mathbf{u}_1 = \mathbf{T}_1\mathbf{v}_1 = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{bmatrix} 0.6 & 0.8 & 0 & 0 \\ 0 & 0 & 0.6 & 0.8 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 2.581\times10^{-5} \\ 1.296\times10^{-5} \end{Bmatrix} = \begin{Bmatrix} ??? \\ ??? \end{Bmatrix} \text{m}$$

Local Force Vector

$$\mathbf{Q}_1 = \mathbf{k}_1\mathbf{u}_1 = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\mathbf{u}_1 = 4\times10^5\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} ??? \\ ??? \end{Bmatrix} = \begin{Bmatrix} ??? \\ ??? \end{Bmatrix}\text{kN}$$

The axial force in member1: N_1= ??? kN (???)

Global Force Vector

$$\mathbf{F}_1 = \mathbf{T}_1^{\mathrm{T}}\mathbf{Q}_1 = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} 0.6 & 0 \\ 0.8 & 0 \\ 0 & 0.6 \\ 0 & 0.8 \end{bmatrix}\begin{Bmatrix} ??? \\ ??? \end{Bmatrix} = \begin{Bmatrix} \boxed{\begin{matrix} ??? \\ ??? \end{matrix}} \\ ??? \\ ??? \end{Bmatrix}\text{kN}$$

$$\mathbf{R}_1 = \begin{Bmatrix} ???\,\text{kN} \\ ???\,\text{kN} \end{Bmatrix}$$

Reactions at Support 1

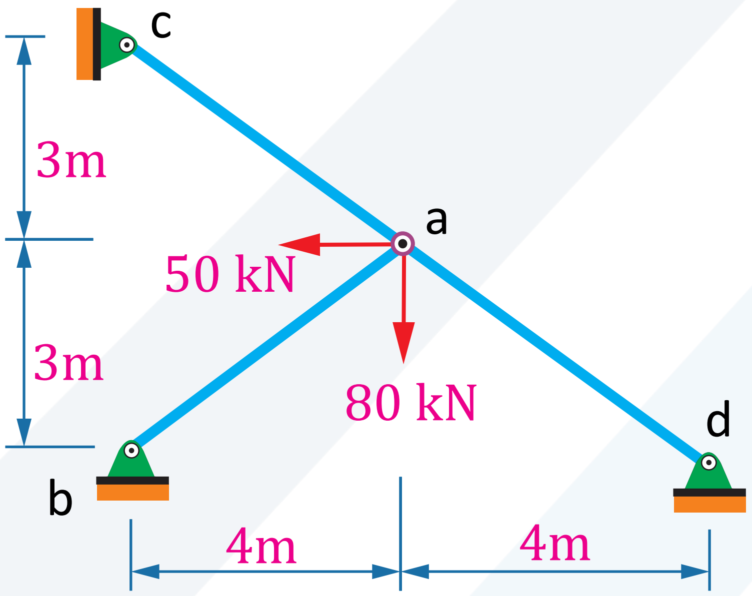
Example 2

For the truss shown, use the stiffness method to:

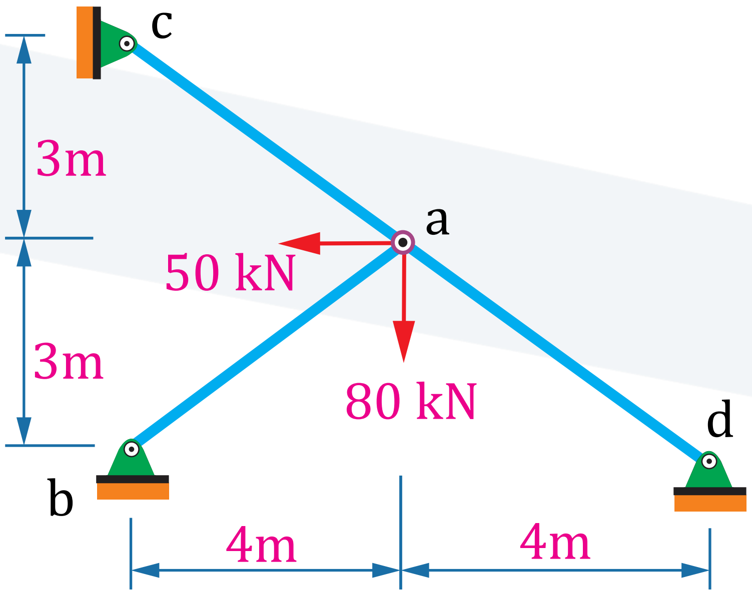
(a) Determine the deflections of the loaded joint.

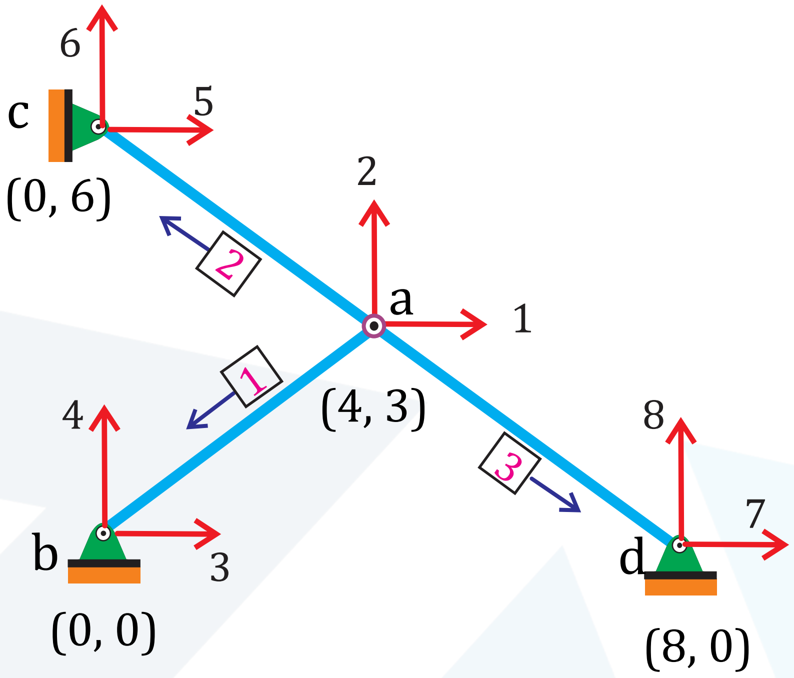
(b) Determine the end forces of each member and reactions at supports.

Assume *EA* to be the same for each member



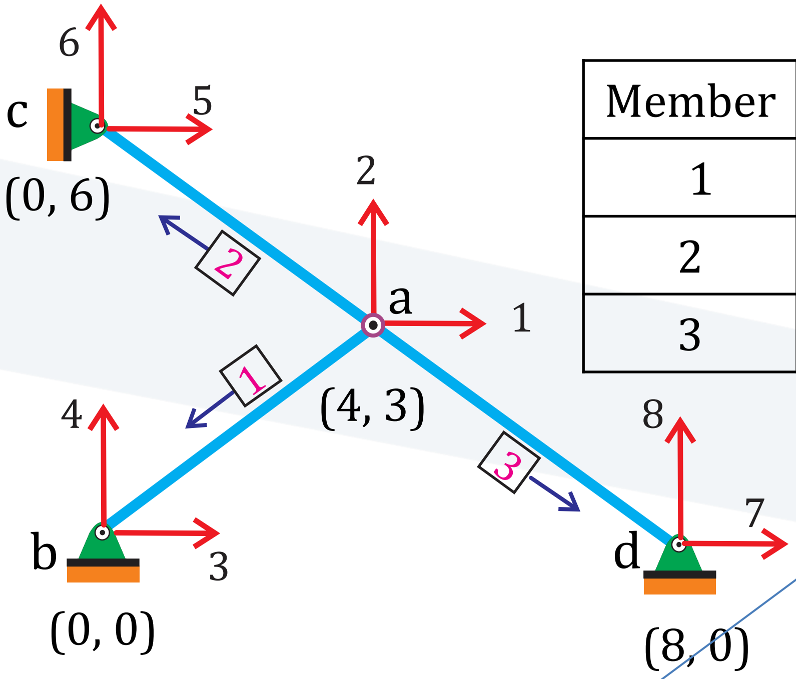
Global element stiffness matrix





Member	c	s
1	$(0-4)/5=-0.8$	$(0-3)/5=-0.6$
2	$(0-4)/5=-0.8$	$(6-3)/5=0.6$
3	$(8-4)/5=0.8$	$(0-3)/5=-0.6$

B. Haidar

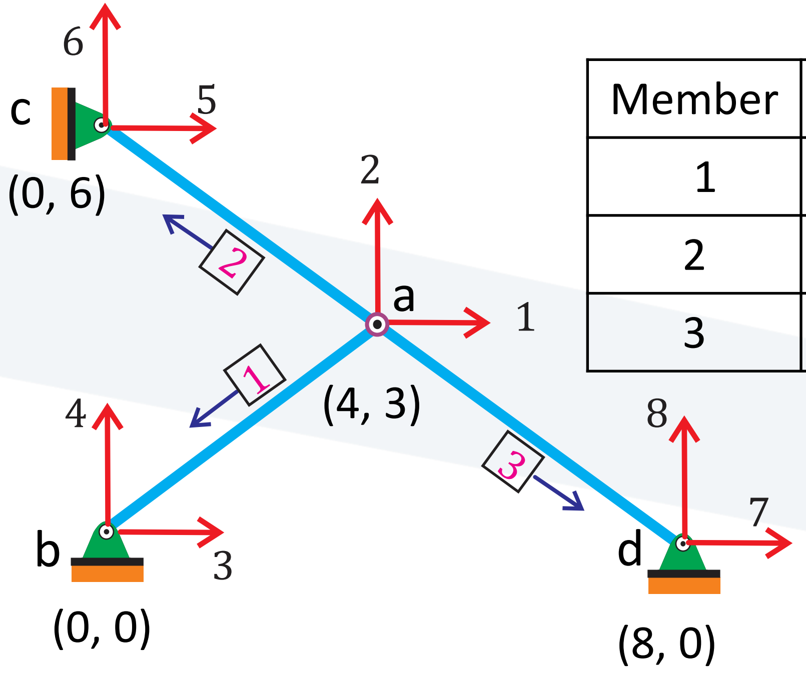


Member	c	s	c²	cs	s²
1	-0.8	-0.6	0.64	0.48	0.36
2	-0.8	0.6	0.64	-0.48	0.36
3	0.8	-0.6	0.64	-0.48	0.36

$$[K] = \frac{EA}{L} \begin{array}{c} \begin{array}{cccc} \Delta_{xi} & \Delta_{yi} & \Delta_{xj} & \Delta_{yj} \end{array} \\ \left[\begin{array}{cc|cc} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ \hline -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{array}\right] \end{array} \begin{array}{c} F_{xi} \\ F_{yi} \\ F_{xj} \\ F_{yj} \end{array}$$

$$[K]_1 = \frac{EA}{5} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cc|cc} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ \hline -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}$$

$a \rightarrow b$



Member	c	s	c^2	cs	s^2
1	-0.8	-0.6	0.64	0.48	0.36
2	-0.8	0.6	0.64	-0.48	0.36
3	0.8	-0.6	0.64	-0.48	0.36

$$[K]_2 = \frac{EA}{5} \quad a \rightarrow c$$

$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ & \left[\begin{array}{cc|cc} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ \hline -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{array}\right] & \begin{array}{c} 1 \\ 2 \\ 5 \\ 6 \end{array} \end{array}$$

$$[K]_3 = \frac{EA}{5} \quad a \rightarrow d$$

$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ & \left[\begin{array}{cc|cc} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ \hline -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{array}\right] & \begin{array}{c} 1 \\ 2 \\ 7 \\ 8 \end{array} \end{array}$$

$$[K]_1 = \frac{EA}{5}\begin{array}{c} \begin{matrix} 1 & \quad 2 & \quad 3 & \quad 4 \end{matrix} \\ \left[\begin{array}{cc|cc} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ \hline -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{array}\right] \end{array} \begin{matrix} \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

$$[K]_2 = \frac{EA}{5}\begin{array}{c} \begin{matrix} 1 & \quad 2 & \quad 5 & \quad 6 \end{matrix} \\ \left[\begin{array}{cc|cc} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ \hline -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{array}\right] \end{array} \begin{matrix} \\ 1 \\ 2 \\ 5 \\ 6 \end{matrix}$$

$$[K]_3 = \frac{EA}{5}\begin{array}{c} \begin{matrix} 1 & \quad 2 & \quad 7 & \quad 8 \end{matrix} \\ \left[\begin{array}{cc|cc} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ \hline -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{array}\right] \end{array} \begin{matrix} \\ 1 \\ 2 \\ 7 \\ 8 \end{matrix}$$

$$[K_{ff}] = S = \frac{EA}{5}\begin{array}{c} \begin{matrix} 1 & \quad 2 \end{matrix} \\ \left[\begin{array}{cc} 1.92 & -0.48 \\ -0.48 & 1.08 \end{array}\right] \end{array} \begin{matrix} \\ 1 \\ 2 \end{matrix}$$

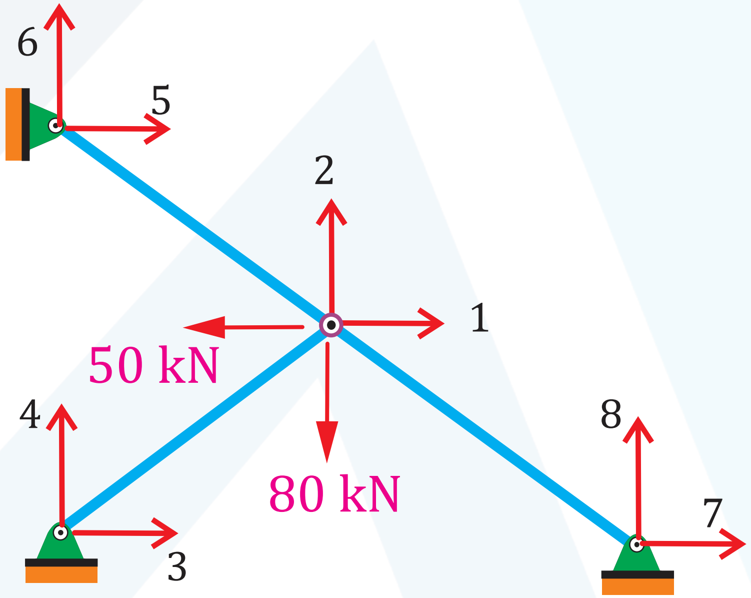
$$\{F_f\} = [K_{ff}]\{\Delta_f\} \qquad \bar{\mathbf{P}} = \mathbf{S}\mathbf{d}$$

$$\begin{array}{c} 1 \\ 2 \end{array}\begin{Bmatrix} F_1 = -50 \\ F_2 = -80 \end{Bmatrix} = \frac{EA}{5} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 1.92 & -0.48 \\ -0.48 & 1.08 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix}$$

$$-50 = \frac{EA}{5}(1.92\,\Delta_1 - 0.48\,\Delta_2) \quad \text{..........} \ (1)$$

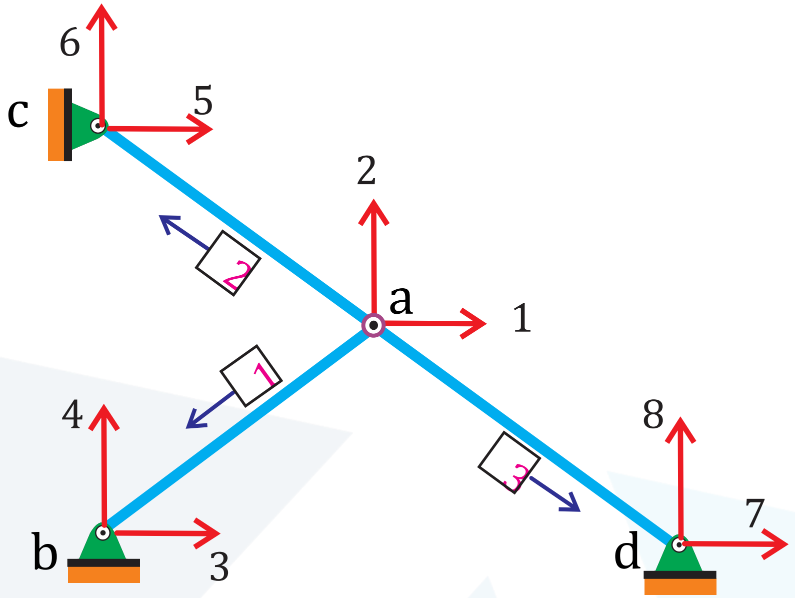
$$-80 = \frac{EA}{5}(-0.48\,\Delta_1 + 1.08\,\Delta_2) \quad \text{..........} \ (2)$$

$$\begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} = \begin{Bmatrix} -250.65/EA \\ -481.77/EA \end{Bmatrix}$$



Member forces

Member	c	s
1	−0.8	−0.6
2	−0.8	0.6
3	0.8	−0.6



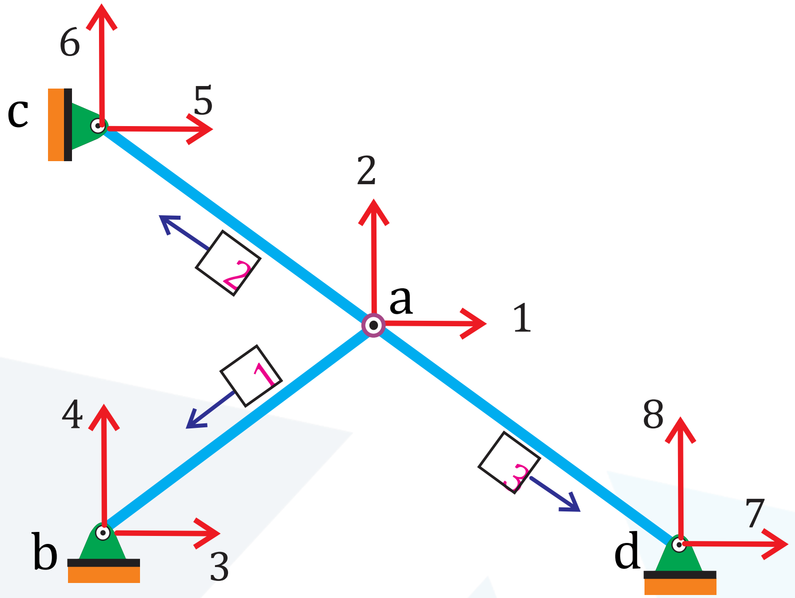
$$f_j = \frac{EA}{L}\begin{bmatrix}-c & -s & c & s\end{bmatrix}\begin{Bmatrix}\Delta_{ix}\\ \Delta_{iy}\\ \Delta_{jx}\\ \Delta_{jy}\end{Bmatrix}$$

Member 1 (a-b)

$$\{F\}_1 = \frac{EA}{5}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix}0.8 & 0.6 & -0.8 & -0.6\end{bmatrix} \end{array}\begin{Bmatrix}\Delta_1 = -250.65/EA\\ \Delta_2 = -481.77/EA\\ \Delta_3 = 0\\ \Delta_4 = 0\end{Bmatrix} = -97.9\ kN$$

compression

Member	c	s
1	−0.8	−0.6
2	−0.8	0.6
3	0.8	−0.6



$$f_j = \frac{EA}{L}\begin{bmatrix} -c & -s & c & s \end{bmatrix}\begin{Bmatrix} \Delta_{ix} \\ \Delta_{iy} \\ \Delta_{jx} \\ \Delta_{jy} \end{Bmatrix}$$

Member 2 (a-c)

$$\{F\}_2 = \frac{EA}{5}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{bmatrix} 0.8 & -0.6 & -0.8 & 0.6 \end{bmatrix} \end{array}\begin{Bmatrix} \Delta_1 = -250.65/EA \\ \Delta_2 = -481.77/EA \\ \Delta_5 = 0 \\ \Delta_6 = 0 \end{Bmatrix} = +17.7\ kN$$

tension

Member 3 (a-d)

$$\{F\}_3 = \frac{EA}{5}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ \begin{bmatrix} -0.8 & 0.6 & 0.8 & -0.6 \end{bmatrix} \end{array}\begin{Bmatrix} \Delta_1 = -250.65/EA \\ \Delta_2 = -481.77/EA \\ \Delta_7 = 0 \\ \Delta_8 = 0 \end{Bmatrix} = -17.7\ kN$$

compression